MINET 2007

Programming (Prelims)

School: _____

Participant 1: _____ Class: _____

Participant 2: _____ Class: _____

Instructions

1. *This paper contains 3 printed pages with 9 questions.*
2. *Please fill in the details asked above before beginning the paper.*
3. *There is **no negative marking**.*
4. *In case of a tie, the questions marked with an asterisk (*) will be tie-breakers.*

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1. Find the number of integral solutions to the following equation.
   1/x + 1/y = 1/12

   Ans: 13 solutions. The equation may be written as (x -12)(y-12) = 144. Now any decomposition of the RHS into two factors results in a solution of the equation. For example 144 = 8*18, thus x-12 = 8 and y-12 =18 gives x=20 and y=30. In all, there are 13 ways to decompose 144 into product of factors.

2. * The four people in this puzzle all competed in different classes of dog agility at a recent competition. The competitions all required the dogs to run over jumps, through tunnels and various other obstacles as quickly as possible. Each had a different result - one came first, one third, one fourth and one ninth. All four dogs were each of a different breed.

   Can you work out who handled which dog, at what level each competed, the place each finished in and the breed of each dog?

a) If Tiff finished first then Terry finished fourth.
b) If Terry finished fourth then Jago is a collie otherwise Jago is not a collie.
c) If Jane competed in the Senior class then she finished third.
d) If Jane competed in Novice then she finished fourth.
e) The dog that finished ninth was an alsatian. This was either Jago, in which case Jago competed in the Elementary class, or this was Kelly, in which case Terry handled Kelly.
f) Mark won Starters.
g) If Mark's dog is called Patti then Patti is a labrador otherwise Patti is a collie.
h) Ruth's dog is called Jago.
i) If Jago finished fourth then she competed in the Novice class otherwise she competed in the Senior class.
j) If Patti finished first then Terry's dog is an alsatian otherwise Terry's dog is a collie.
k) If Jane's dog is a doberman then Jane finished fourth otherwise Jane finished third.

   Handler's Names: Jane, Mark, Ruth and Terry
   Dog's Names: Tiff, Patti, Jago and Kelly
   Breed: Alsatian, Collie, Labrador and Doberman
   Level: Starters, Elementary, Novice or Senior

Ans:

Dog	Handler	Class	Place	Breed
Tiff	Jane	Senior	3rd	Collie
Patti	Mark	Starters	1st	Labrador
Jago	Ruth	Novice	4th	Doberman
Kelly	Terry	Elementary	9th	Alsatian

3. Which is the next higher permuation of 679385421?

   Ans: 679412358

4. A Buddhist monk got an errand from his teacher: to meditate for exactly 45 minutes. He has no watch; instead he is given two incense sticks, and he is told that each of those sticks would completely burn in 1 hour. The sticks are not identical, and they burn with variant yet unknown rates (they are hand-made). So he has these two incense and some matches: can he arrange for exactly 45 minutes of meditation?

   Ans: Burn one of the sticks from both sides and the other from one side; wait for the first to burn out. As soon as it burns out, light the other end of the second stick and wait for it to burn out as well.

5. Fermat's last theorem states that there cannot be integers (x, y, z, N > 2) such that
   x^N + y^N = z^N

   It was one of the most alluring of mathematical statements whose proof eluded mathematicians for many years before it was proved by Andrew Wiles in the 1990s. A computer scientist claims that he proved somehow that the Fermat theorem is correct for the following 3 numbers:

   x=2233445566,
   y=7788990011,
   z=9988776655

   He announces these 3 numbers and calls for a press conference where he is going to present the value of N and to show that
   x^N + y^N = z^N

   As the press conference starts, a 10-year old boy raises his hand and says that the respectable scientist has made a mistake and the Fermat theorem cannot hold for those 3 numbers. The scientist checks his computer calculations and finds a bug.

   How did the boy figure out that the scientist was wrong?

   Ans: The smart boy looked at only the last digits of the numbers. Any power of a number ending in 6 will have 6 as the last digit and the same can be said about 5 and 1. Now notice that the last digits don't really add up in Fermat's equation.

6. Two blondes are sitting in a street cafe, talking about the children. One says that she has three daughters. The product of their ages equals 36 and the sum of the ages coincides with the number of the house across the street. The second blonde replies that this information is not enough to figure out the age of each child. The first agrees and adds that the oldest daughter has the beautiful blue eyes. Then the second solves the puzzle. You might solve it too!

   Ans: (2,2,9). There are a number of possible ways to represent 36 as the product of three integers, however only two of them are such that the sum of the ages is same {(2,2,9) and (1,6,6)}. The solution is one of these, because had

this not been the case then the second blonde would have counted the number of houses across the street and found the answer. From the second statement we know that there is a unique oldest child among her children, so (1,6,6) is ruled out.

7. Using four 4s and any mathematical operation generate 224. If you are using exponentiation/log then generate the required bases and exponents. i.e. sqrt(4) is not allowed.

   Ans: (4 + 4!)*(4 + 4)

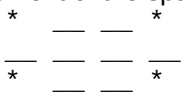
8. A man had to pack a sack of apples into packets but as each packet had to have exactly the same number of apples he was having difficulty. If he packed 10 apples per packet, one packet only had 9. If he packed 9 apples per packet, one packet only had 8. If he packed 8 apples per packet, one packet only had 7. If he packed 7 apples per packet, one packet only had 6. And so on down to 2 apples. How many apples did he start with?

   Ans: 2519

9. Fill the eight spaces below with numbers from 1 to 8. No number that comes before or after any other number should be placed in an adjacent space, either horizontally, vertically or diagonally.

   (For example 2 should not be next to the number 1 or the number 3)

   The arrangement of the space is

   Ans:
   *** 3 5 ***
   7 1 8 2
   *** 4 6 ***